International Journal of Humanities Social Science and Management (IJHSSM)
Volume 4, Issue 6, Nov.-Dec., 2024, pp: 629-636 ISSN: 3048-6874
www.ijhssm.org

# Fuzzy Inventory Model with Various Defuzzification Methods Using Kuhn Tucker Technique

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Date of Submission: 16-11-2024 Date of Acceptance: 01-12-2024

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**Abstract:** In this paper, according to a rework approach, the defective items were categorized as synchronous and asynchronous. At the end of the procedure, the rework technique involves sorting and gathering the flawed components. This paper takes into account asynchronous faulty products and attempted to reduce the total cost. Using pentagonal fuzzy numbers, parameters are fuzzified. Signed distance, Pascal triangular graded mean method and Graded Mean integration representation methods are used for defuzzification. Optimal Solution is obtained using Kuhn Tucker technique. Finally, Numerical results are analyzed and compared.

**Keywords:** Pentagonal Fuzzy number, Pascal triangular graded mean method, Signed distance Method, Graded mean integration representation method, Kuhn Tucker Method.

## I. Introduction

In the field of applied mathematics, an objective function can be maximized and minimized in optimization technique. In the subject of inventory management, the objective of optimization problems is crucial. The goal is to reduce total cost, which is made up of several variable factors like ordering cost, holding cost and shortages. One of the crucial challenges to reduce costs and increase profit is inventory management. Recently, Classical inventory model has been treated by using fuzzy set theory. Zadeh (1965) first introduced the fuzzy set concept. Advances made in the area of fuzzy and its applications followed. Harris (1913) first presented the Economic Order Quantity (EOQ) model. Chang(1999) determined economic production quantity when the demand is uncertain. Park(1987) discussed fuzzy sets in Economic Order quantity. Demand and order quantity were treated as crisp sense while order cost and holding cost were treated as fuzzy parameters M. Vujosevic(1996) applied EOQ formula for the fuzzy inventory cost

Hill and Omar(2006) discussed that the supply chain participants might achieve synchronization while minimizing operational costs and halving benefits by jointly managing the production and scheduling policies. Lavanya P(2017) given various fuzzy numbers and their various ranking method. Rama(2018) compared among with various defuzzification techniques of various fuzzy numbers in both crisp and fuzzy sense. Kalaiarasi(2022) derived optimal order quantity with various fuzzy components by using Lagrangian method.

In this paper, Section 2 the mathematical formulation of inventory model is consider along with its notation. Section 3 parameters are fuzzified and pentagonal fuzzy number used to solve the fuzzy inventory model with various defuzzification method. Section 4 numerical examples is analyzed and tabulates the results of computing the hexagonal fuzzy numbers using three defuzzification techniques.

## II. Definitions and Methodologies

i. **Fuzzy set:** A fuzzy set $\tilde{A}_\alpha$in X is defined as the following set of ordered pairs
$\tilde{A}_\alpha = \{\left(x, \mu_{\tilde{A}_\alpha}(x)\right) : x\epsilon X\}$. The membership function of $x\epsilon X$ in $\tilde{A}_\alpha$ is the mapping $\mu_{\tilde{A}_\alpha} : X \to [0, 1]$.

ii. **The Function Principle:** Suppose $\tilde{E} = (e_1, e_2, e_3, e_4, e_5)$ *and* $\tilde{F} = (f_1, f_2, f_3, f_4, f_5)$ are two pentagonal fuzzy numbers then,

- The addition of $\tilde{E}$ and $\tilde{F}$ is $\tilde{E} \oplus \tilde{F} = (e_1 + f_1, e_2 + f_2, e_3 + f_3, e_4 + f_4, e_5 + f_5)$

-  The multiplication of $\tilde{E}$ and $\tilde{F}$ is $\tilde{E} \otimes \tilde{F} = (e_1 f_1,\ e_2 f_2,\ e_3 f_3, e_4 f_4,\ e_5 f_5)$
-  The subtraction of $\tilde{E}$and $\tilde{F}$ is $\tilde{E} \ominus \tilde{F} = (e_1 - f_5, e_2 - f_4, e_3 - f_3, e_4 - f_2, e_5 - f_1)$
-  The division of $\tilde{E}$ and $\tilde{F}$ is $\frac{\tilde{E}}{\tilde{F}} = \left(\frac{e_1}{f_5}, \frac{e_2}{f_4}, \frac{e_3}{f_3}, \frac{e_4}{f_2}, \frac{e_5}{f_1}\right)$
-  Let $\alpha \epsilon$R, Then $\alpha \geq 0$ , $\alpha \otimes \tilde{E} = (\alpha e_1, \alpha e_2, \alpha e_3, \alpha e_4, \alpha e_5)$
  $\alpha < 0$ , $\alpha \otimes \tilde{F} = (\alpha f_1, \alpha f_2, \alpha f_3, \alpha f_4, \alpha f_5)$

iii. **Pentagonal Fuzzy Number:** A Pentagonal Fuzzy Number $\tilde{U} = (u_{1,}\ u_{2,} u_{3,} u_4, u_5)$ is represented with membership function

$$\mu_{\tilde{U}}(x) = \begin{cases} L_1(x) = \left(\dfrac{x - u_1}{u_2 - u_1}\right), u_1 \leq x \leq u_2 \\ L_2(x) = \left(\dfrac{x - u_2}{u_3 - u_2}\right), u_2 \leq x \leq u_3 \\ R_1(x) = \left(\dfrac{u_4 - x}{u_4 - u_3}\right), x_3 \leq x \leq x_4 \\ R_2(x) = \left(\dfrac{u_5 - x}{u_5 - u_4}\right), x_4 \leq x \leq x_5 \\ 0, otherwise \end{cases}$$

iv. **Pascal Triangular Graded Mean for Pentagonal Fuzzy Number:**
Pentagonal Fuzzy number $\tilde{U} = (u_{1,}\ u_{2,} u_{3,} u_4, u_5) is\ defined\ as$

$$P(\tilde{U}) = \frac{u_1 + 4u_2 + 6u_3 + 4u_4 + u_5}{16}$$

v. **Signed Distance Method:**
Let $\tilde{U}$ be fuzzy set defined on R, then the signed distance of $\tilde{U}$ is defined as

$$P(\tilde{U}) = \frac{u_1 + 2u_2 + 2u_3 + 2u_4 + u_5}{8}$$

vi. **Graded Mean Integration Representation Method :**
The Graded Mean Integration Representation of $\tilde{U}$ is defined as

$$P(\tilde{U}) = \frac{u_1 + 3u_2 + 4u_3 + 3u_4 + u_5}{12}$$

**Notations:**

$Q$ - Order quantity
$\beta$ - Annual cost of capital investment
$R(\beta)$ - Investment to decrease the lost sales
$S$ - Holding Cost/unit/year
$E$ - Stock out
$C(u - w)$- Expected Shortages at the end of the cycle
$L$ - Leadtime Crashing Cost
$X$ - Lead time/week
$F$ - Safety factor
$H$ - Demand(units/year)

**Mathematical Model in Crisp Sense:**

We deal an inventory model in crisp sense and Economic Order quantity can be obtained by the following equation

$$T_c = \beta R(\beta) + \frac{1}{Q}[E + L] + S\left[F - HX + \frac{HQ}{2} + \beta C(u - w)\right] \quad \text{-------------- (1)}$$

Partially differentiating equation (1) with respect to Q and equate to 0, we get

$$\frac{\partial T_c}{\partial Q} = 0 \Rightarrow -\frac{1}{Q^2}[E+L] + \frac{SH}{2} = 0$$

Hence, the economic order quantity is given by

$$\Rightarrow Q^* = \sqrt{\frac{2[E+L]}{SH}} \qquad \text{-------------- (2)}$$

## III. Mathematical Model in Fuzzy Sense:

To consider the mathematical model in fuzzy sense, The parameters 'E, 'L', 'S' AND 'H' are fuzzified using pentagonal fuzzy number and

I. Defuzzification using Pascal triangular graded mean method is applied

$$\tilde{E} = (e_1, e_2, e_3, e_4, e_5), \tilde{L} = (l_1, l_2, l_3, l_4, l_5)\ \tilde{S} = (s_1, s_2, s_3, s_4, s_5), and \tilde{H} = (h_1, h_2, h_3, h_4, h_5)$$

Now, the fuzzy total cost can be arrived from equation (1) as follows:

$$\tilde{T_c} = \beta R(\beta) + \frac{1}{Q}[\tilde{E}+\tilde{L}] + \tilde{S}\left[F - \tilde{H}X + \frac{\tilde{H}Q}{2} + \beta C(u-w)\right]$$

applying the function principle on Pentagonal fuzzy numbers, we get

$$\tilde{T_c} = \beta R(\beta) + \frac{1}{Q}[e_1+l_1] + s_1\left[F - h_1X + \frac{h_1Q}{2} + \beta C(u-w)\right],$$
$$\beta R(\beta) + \frac{1}{Q}[e_2+l_2] + s_2\left[F - h_2X + \frac{h_2Q}{2} + \beta C(u-w)\right],$$
$$\beta R(\beta) + \frac{1}{Q}[e_3+l_3] + s_3\left[F - h_3X + \frac{h_3Q}{2} + \beta C(u-w)\right],$$
$$\beta R(\beta) + \frac{1}{Q}[e_4+l_4] + s_4\left[F - h_4X + \frac{h_4Q}{2} + \beta C(u-w)\right],$$
$$\beta R(\beta) + \frac{1}{Q}[e_5+l_5] + s_5\left[F - h_5X + \frac{h_5Q}{2} + \beta C(u-w)\right] \qquad \text{----------- (3)}$$

By Pascal triangular graded mean method, we get

$$P(\tilde{T_c}) = \frac{1}{16}\Bigg[\left(\beta R(\beta) + \frac{1}{Q}[e_1+l_1] + s_1\left[F - h_1X + \frac{h_1Q}{2} + \beta C(u-w)\right]\right)$$
$$+ 4\left(\beta R(\beta) + \frac{1}{Q}[e_2+l_2] + s_2\left[F - h_2X + \frac{h_2Q}{2} + \beta C(u-w)\right]\right)$$
$$+ 6\left(\beta R(\beta) + \frac{1}{Q}[e_3+l_3] + s_3\left[F - h_3X + \frac{h_3Q}{2} + \beta C(u-w)\right]\right)$$
$$+ 4\left(\beta R(\beta) + \frac{1}{Q}[e_4+l_4] + s_4\left[F - h_4X + \frac{h_4Q}{2} + \beta C(u-w)\right]\right)$$
$$+ \left(\beta R(\beta) + \frac{1}{Q}[e_5+l_5] + s_5\left[F - h_5X + \frac{h_5Q}{2} + \beta C(u-w)\right]\right)\Bigg] \qquad -----(4)$$

Partially differentiate the equation (4) with respect to Q and equate it to zero.

$$\frac{\partial P(\tilde{T_c})}{\partial Q} = 0 \Rightarrow Q = \sqrt{\frac{2[(e_1+l_1)+4(e_2+l_2)+6(e_3+l_3)+4(e_4+l_4)+(e_5+l_5)]}{s_1h_1+4s_2h_2+6s_3h_3+4s_4h_4+s_5h_5}} \qquad \text{---------- (5)}$$

Hence, the equation (4) is the fuzzy total cost and the equation (5) is the fuzzy economic quantity

**KUHN TUCKER METHOD**

The Kuhn-Tucker Condition is,

(i) $\lambda \le 0$
(ii) $\nabla f p(T_C) - \lambda \nabla g(Q) = 0$
(iii) $\lambda_i g_i(Q) = 0$, i = 1, 2, … m
(iv) $g_i(Q) \ge 0, i = 1, 2, \dots, m$

**OPTIMAL SOLUTIONS OBTAINED BY KUHN TUCKER METHOD**

Suppose fuzzy order quantity $\tilde{Q} = (q_1, q_2, q_3, q_4, q_5)$

By function principle, the fuzzy total cost is

$$\tilde{T_c} = \beta R(\beta) + \frac{1}{q_5}[e_1+l_1] + s_1\left[F - h_1X + \frac{h_1q_1}{2} + \beta C(u-w)\right],$$
$$\beta R(\beta) + \frac{1}{q_4}[e_2+l_2] + s_2\left[F - h_2X + \frac{h_2 q_2}{2} + \beta C(u-w)\right],$$

$$\beta R(\beta)+\frac{1}{q_3}[e_3+l_3]+s_3\left[F-h_3X+\frac{h_3q_3}{2}+\beta C(u-w)\right],$$
$$\beta R(\beta)+\frac{1}{q_2}[e_4+l_4]+s_4\left[F-h_4X+\frac{h_4q_4}{2}+\beta C(u-w)\right],$$
$$\beta R(\beta)+\frac{1}{q_1}[e_5+l_5]+s_5\left[F-h_5X+\frac{h_5q_5}{2}+\beta C(u-w)\right] \quad \text{------------- (6)}$$

Defuzzification the fuzzy total cost by using Pascal triangular graded mean method

$$P(\widetilde{T_c})=\frac{1}{16}\Bigg[\left(\beta R(\beta)+\frac{1}{q_5}[e_1+l_1]+s_1\left[F-h_1X+\frac{h_1q_1}{2}+\beta C(u-w)\right]\right)$$
$$+4\left(\beta R(\beta)+\frac{1}{q_4}[e_2+l_2]+s_2\left[F-h_2X+\frac{h_2q_2}{2}+\beta C(u-w)\right]\right)$$
$$+6\left(\beta R(\beta)+\frac{1}{q_3}[e_3+l_3]+s_3\left[F-h_3X+\frac{h_3q_3}{2}+\beta C(u-w)\right]\right)$$
$$+4\left(\beta R(\beta)+\frac{1}{q_2}[e_4+l_4]+s_4\left[F-h_4X+\frac{h_4q_4}{2}+\beta C(u-w)\right]\right)$$
$$+\left(\beta R(\beta)+\frac{1}{q_1}[e_5+l_5]+s_5\left[F-h_5X+\frac{h_5q_5}{2}+\beta C(u-w)\right]\right)\Bigg] --------(7)$$

With $0\le q_1\le q_2\le q_3\le q_4\le q_5$
It can be written as $q_2-q_1\ge 0, q_3-q_2\ge 0, q_4-q_3\ge 0, q_5-q_4\ge 0, q_1\ge 0.$

**Condition 1:** $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5\le 0$

**Condition 2:** Partially Differentiate $q_1, q_2, q_3, q_4, q_5$ equation (7) and equate them to zero

$$q_1\Rightarrow\frac{1}{16}\left[\frac{1}{{q_1}^2}[e_5+l_5]+\frac{h_1s_1}{2}\right]+\lambda_1-\lambda_5=0\,;\, q_2\Rightarrow\frac{1}{16}\left[\frac{1}{{q_2}^2}[e_5+l_5]+\frac{h_1s_1}{2}\right]-\lambda_1+\lambda_2=0$$

$$q_3\Rightarrow\frac{1}{16}\left[\frac{1}{{q_3}^2}[e_5+l_5]+\frac{h_1s_1}{2}\right]-\lambda_2+\lambda_3=0\,;\, q_4\Rightarrow\frac{1}{16}\left[\frac{1}{{q_4}^2}[e_5+l_5]+\frac{h_1s_1}{2}\right]-\lambda_3+\lambda_4=0$$

$$q_5\Rightarrow\frac{1}{16}\left[\frac{1}{{q_5}^2}[e_5+l_5]+\frac{h_1s_1}{2}\right]-\lambda_4=0$$

**Condition 3:** $\lambda_1(q_2-q_1)=0, \lambda_2(q_3-q_2)=0, \lambda_3(q_4-q_3)=0, \lambda_4(q_5-q_4)=0, \lambda_5 q_1=0$

**Condition 4:** $q_2-q_1\ge 0, q_3-q_2\ge 0, q_4-q_3\ge 0, q_5-q_4\ge 0, q_1\ge 0.$
Here we know that, $q_1\ge 0,\ \lambda_5 q_1=0,\ so\ we\ get\ \lambda_5=0$
Then we replace $q_2\ by\ q_1, q_3\ by\ q_2, q_4\ by\ q_3, q_5\ by\ q_4,$ (i.e) $(q_1=q_2=q_3=q_4=q_5=Q)$
By adding condition 3, we get,

$$Q^*=\sqrt{\frac{2[(e_1+l_1)+4(e_2+l_2)+6(e_3+l_3)+4(e_4+l_4)+(e_5+l_5)]}{s_1h_1+4s_2h_2+6s_3h_3+4s_4h_4+s_5h_5}}$$

## IV. Signed distance Method

Defuzzification using Signed Distance Method is applied
Consider the equation (3) by Signed Distance Method, we get

$$P(\widetilde{T_c})=\frac{1}{8}\Bigg[\left(\beta R(\beta)+\frac{1}{Q}[e_1+l_1]+s_1\left[F-h_1X+\frac{h_1Q}{2}+\beta C(u-w)\right]\right)$$
$$+2\left(\beta R(\beta)+\frac{1}{Q}[e_2+l_2]+s_2\left[F-h_2X+\frac{h_2Q}{2}+\beta C(u-w)\right]\right)$$
$$+2\left(\beta R(\beta)+\frac{1}{Q}[e_3+l_3]+s_3\left[F-h_3X+\frac{h_3Q}{2}+\beta C(u-w)\right]\right)$$
$$+2\left(\beta R(\beta)+\frac{1}{Q}[e_4+l_4]+s_4\left[F-h_4X+\frac{h_4Q}{2}+\beta C(u-w)\right]\right)$$
$$+\left(\beta R(\beta)+\frac{1}{Q}[e_5+l_5]+s_5\left[F-h_5X+\frac{h_5Q}{2}+\beta C(u-w)\right]\right)\Bigg] ------(8)$$

Partially differentiate the equation (8) with respect to Q and equate it to zero.

$$\frac{\partial P\,(\widetilde{T_c})}{\partial Q} = 0 \Rightarrow Q = \sqrt{\frac{2[(e_1+l_1)+2(e_2+l_2)+2(e_3+l_3)+2(e_4+l_4)+(e_5+l_5)]}{s_1h_1+2s_2h_2+2s_3h_3+2s_4h_4+s_5h_5}} \quad \text{----------} (9)$$

Hence, the equation (8) is the fuzzy total cost and the equation (9) is the fuzzy economic quantity

**OPTIMAL SOLUTIONS OBTAINED BY KUHN TUCKER METHOD**

Suppose fuzzy order quantity $\tilde{Q} = (q_1, q_2, q_3, q_4, q_5)$

Consider the equation (3), Defuzzify the fuzzy total cost by using Signed Distance Method

$$\begin{aligned} P(\widetilde{T_c}) = \frac{1}{8}\Bigg[&\left(\beta R(\beta) + \frac{1}{q_5}[e_1 + l_1] + s_1\left[F - h_1X + \frac{h_1q_1}{2} + \beta C(u-w)\right]\right) \\ &+ 2\left(\beta R(\beta) + \frac{1}{q_4}[e_2 + l_2] + s_2\left[F - h_2X + \frac{h_2q_2}{2} + \beta C(u-w)\right]\right) \\ &+ 2\left(\beta R(\beta) + \frac{1}{q_3}[e_3 + l_3] + s_3\left[F - h_3X + \frac{h_3q_3}{2} + \beta C(u-w)\right]\right) \\ &+ 2\left(\beta R(\beta) + \frac{1}{q_2}[e_4 + l_4] + s_4\left[F - h_4X + \frac{h_4q_4}{2} + \beta C(u-w)\right]\right) \\ &+ \left(\beta R(\beta) + \frac{1}{q_1}[e_5 + l_5] + s_5\left[F - h_5X + \frac{h_5q_5}{2} + \beta C(u-w)\right]\right)\Bigg] \quad ---- \\ &--(10) \end{aligned}$$

With $0 \leq q_1 \leq q_2 \leq q_3 \leq q_4 \leq q_5$

It can be written as $q_2 - q_1 \geq 0, q_3 - q_2 \geq 0, q_4 - q_3 \geq 0, q_5 - q_4 \geq 0, q_1 \geq 0.$

**Condition 1:** $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \leq 0$

**Condition 2:** Partially Differentiate $q_1, q_2, q_3, q_4, q_5$ equation(10) and equate them to zero

$$q_1 \Rightarrow \frac{1}{8}\left[\frac{1}{{q_1}^2}[e_5 + l_5] + \frac{h_1s_1}{2}\right] + \lambda_1 - \lambda_5 = 0\,; q_2 \Rightarrow \frac{1}{8}\left[\frac{1}{{q_2}^2}[e_5 + l_5] + \frac{h_1s_1}{2}\right] - \lambda_1 + \lambda_2 = 0$$

$$q_3 \Rightarrow \frac{1}{8}\left[\frac{1}{{q_3}^2}[e_5 + l_5] + \frac{h_1s_1}{2}\right] - \lambda_2 + \lambda_3 = 0\,;\ q_4 \Rightarrow \frac{1}{8}\left[\frac{1}{{q_4}^2}[e_5 + l_5] + \frac{h_1s_1}{2}\right] - \lambda_3 + \lambda_4 = 0$$

$$q_5 \Rightarrow \frac{1}{8}\left[\frac{1}{{q_5}^2}[e_5 + l_5] + \frac{h_1s_1}{2}\right] - \lambda_4 = 0$$

**Condition 3:** $\lambda_1(q_2 - q_1) = 0, \lambda_2(q_3 - q_2) = 0, \lambda_3(q_4 - q_3) = 0, \lambda_4(q_5 - q_4) = 0, \lambda_5 q_1 = 0$

**Condition 4:** $q_2 - q_1 \geq 0, q_3 - q_2 \geq 0, q_4 - q_3 \geq 0, q_5 - q_4 \geq 0, q_1 \geq 0.$

Here we know that, $q_1 \geq 0,\ \lambda_5 q_1 = 0,\ so\ we\ get\ \lambda_5 = 0$

Then we replace $q_2\ by\ q_1, q_3\ by\ q_2, q_4\ by\ q_3, q_5\ by\ q_4,$ (i.e) $(q_1 = q_2 = q_3 = q_4 = q_5 = Q)$

By adding condition 3, we get,

$$Q^* = \sqrt{\frac{2[(e_1+l_1)+2(e_2+l_2)+2(e_3+l_3)+2(e_4+l_4)+(e_5+l_5)]}{s_1h_1+2s_2h_2+2s_3h_3+2s_4h_4+s_5h_5}}$$

## V. Graded Mean Integration Representation Method

Defuzzification using Graded Mean Integration Representation Method is applied. Consider the equation (3) by Graded Mean Integration Representation Distance Method, we get

**International Journal of Humanities Social Science and Management (IJHSSM)**
**Volume 4, Issue 6, Nov.-Dec., 2024, pp: 629-636 ISSN: 3048-6874**
**www.ijhssm.org**

$$P(\widetilde{T_c}) = \frac{1}{12}\Bigg[\left(\beta R(\beta) + \frac{1}{Q}[e_1 + l_1] + s_1\left[F - h_1 X + \frac{h_1 Q}{2} + \beta C(u - w)\right]\right)$$
$$+ 3\left(\beta R(\beta) + \frac{1}{Q}[e_2 + l_2] + s_2\left[F - h_2 X + \frac{h_2 Q}{2} + \beta C(u - w)\right]\right)$$
$$+ 4\left(\beta R(\beta) + \frac{1}{Q}[e_3 + l_3] + s_3\left[F - h_3 X + \frac{h_3 Q}{2} + \beta C(u - w)\right]\right)$$
$$+ 3\left(\beta R(\beta) + \frac{1}{Q}[e_4 + l_4] + s_4\left[F - h_4 X + \frac{h_4 Q}{2} + \beta C(u - w)\right]\right)$$
$$+ \left(\beta R(\beta) + \frac{1}{Q}[e_5 + l_5] + s_5\left[F - h_5 X + \frac{h_5 Q}{2} + \beta C(u - w)\right]\right)\Bigg] \quad ----- (11)$$

Partially differentiate the equation (11) with respect to Q and equate it to zero.

$$\frac{\partial P(\widetilde{T_c})}{\partial Q} = 0 \Rightarrow Q = \sqrt{\frac{2[(e_1+l_1)+3(e_2+l_2)+4(e_3+l_3)+3(e_4+l_4)+(e_5+l_5)]}{s_1h_1+3s_2h_2 42s_3h_3+3s_4h_4+s_5h_5}} \quad ---------- (12)$$

Hence, the equation (11) is the fuzzy total cost and the equation (12) is the fuzzy economic quantity

**OPTIMAL SOLUTIONS OBTAINED BY KUHN TUCKER METHOD**

Suppose fuzzy order quantity $\tilde{Q} = (q_1, q_2, q_3, q_4, q_5)$

Consider the equation (3), Defuzzify the fuzzy total cost by using Graded Mean Integration Representation

$$P(\widetilde{T_c}) = \frac{1}{12}\Bigg[\left(\beta R(\beta) + \frac{1}{q_5}[e_1 + l_1] + s_1\left[F - h_1 X + \frac{h_1 q_1}{2} + \beta C(u - w)\right]\right)$$
$$+ 3\left(\beta R(\beta) + \frac{1}{q_4}[e_2 + l_2] + s_2\left[F - h_2 X + \frac{h_2 q_2}{2} + \beta C(u - w)\right]\right)$$
$$+ 4\left(\beta R(\beta) + \frac{1}{q_3}[e_3 + l_3] + s_3\left[F - h_3 X + \frac{h_3 q_3}{2} + \beta C(u - w)\right]\right)$$
$$+ 3\left(\beta R(\beta) + \frac{1}{q_2}[e_4 + l_4] + s_4\left[F - h_4 X + \frac{h_4 q_4}{2} + \beta C(u - w)\right]\right)$$
$$+ \left(\beta R(\beta) + \frac{1}{q_1}[e_5 + l_5] + s_5\left[F - h_5 X + \frac{h_5 q_5}{2} + \beta C(u - w)\right]\right)\Bigg] \quad ------- (10)$$

With $0 \le q_1 \le q_2 \le q_3 \le q_4 \le q_5$

It can be written as $q_2 - q_1 \ge 0, q_3 - q_2 \ge 0, q_4 - q_3 \ge 0, q_5 - q_4 \ge 0, q_1 \ge 0.$

**Condition 1:** $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \le 0$

**Condition 2:** Partially Differentiate $q_1, q_2, q_3, q_4, q_5$ *equation* (10)*and equate them to zero*

$$q_1 \Rightarrow \frac{1}{12}\left[\frac{1}{{q_1}^2}[e_5 + l_5] + \frac{h_1 s_1}{2}\right] + \lambda_1 - \lambda_5 = 0\,; q_2 \Rightarrow \frac{1}{12}\left[\frac{1}{{q_2}^2}[e_5 + l_5] + \frac{h_1 s_1}{2}\right] - \lambda_1 + \lambda_2 = 0$$

$$q_3 \Rightarrow \frac{1}{12}\left[\frac{1}{{q_3}^2}[e_5 + l_5] + \frac{h_1 s_1}{2}\right] - \lambda_2 + \lambda_3 = 0\,;\; q_4 \Rightarrow \frac{1}{12}\left[\frac{1}{{q_4}^2}[e_5 + l_5] + \frac{h_1 s_1}{2}\right] - \lambda_3 + \lambda_4 = 0$$

$$q_5 \Rightarrow \frac{1}{12}\left[\frac{1}{{q_5}^2}[e_5 + l_5] + \frac{h_1 s_1}{2}\right] - \lambda_4 = 0$$

**Condition 3:** $\lambda_1(q_2 - q_1) = 0, \lambda_2(q_3 - q_2) = 0, \lambda_3(q_4 - q_3) = 0, \lambda_4(q_5 - q_4) = 0, \lambda_5 q_1 = 0$

**Condition 4:** $q_2 - q_1 \ge 0, q_3 - q_2 \ge 0, q_4 - q_3 \ge 0, q_5 - q_4 \ge 0, q_1 \ge 0.$

Here we know that, $q_1 \ge 0,\ \lambda_5 q_1 = 0,\ so\ we\ get\ \lambda_5 = 0$

Then we replace $q_2\ by\ q_1, q_3\ by\ q_2, q_4\ by\ q_3, q_5\ by\ q_4,$ (i.e) $(q_1 = q_2 = q_3 = q_4 = q_5 = Q)$

By adding condition 3, we get,

**International Journal of Humanities Social Science and Management (IJHSSM)**
**Volume 4, Issue 6, Nov.-Dec., 2024, pp: 629-636** **ISSN: 3048-6874**
**www.ijhssm.org**

$$Q^* = \sqrt{\frac{2[(e_1+l_1)+3(e_2+l_2)+4(e_3+l_3)+3(e_4+l_4)+(e_5+l_5)]}{s_1h_1+3s_2h_2+4s_3h_3+3s_4h_4+s_5h_5}}$$

**Numerical Example:**

Consider the parametric values as: $E = 80\,; L = 8; S = 6unit/year\,; H = 15\; unit/year$

| Variations | Values (Pentagonal Fuzzy Numbers) | Crisp Value | Fuzzy Value | | |
|---|---|---|---|---|---|
| | | | Pascal Triangular Method | Signed Distance | Graded Mean Method |
| -25% | $E = 60; L = 6; S = 4.5; H = 11.25$<br>$E = (50, 55, 60, 65, 70)$<br>$L = (5, 5.5, 6, 6.5, 7)$<br>$S = (3.5, 4, 4.5, 5, 5.5)$<br>$H = (10.5,11,11.25,11.50,11.75)$ | 1.6147 | 1.6127 | 1.6117 | 1.6124 |
| -50% | $E = 40; L = 4; S = 3; H = 7.5$<br>$E = (30, 35, 40, 45, 50)$<br>$L = (3, 3.5, 4, 4.5, 5)$<br>$S = (1, 2, 3, 4, 5)$<br>$H = (6.5, 7, 7.5, 8, 8.5)$ | 1.9776 | 1.9560 | 1.9455 | 1.9525 |
| 0 | $E = 80; L = 8; S = 6; H = 15$<br>$E = (70,75,80,85,90)$<br>$L = (7,7.5,8,8.5,9)$<br>$S = (4,5,6,7,8)$<br>$H = (13,14,15,16,17)$ | 1.3984 | 1.3907 | 1.3869 | 1.3894 |
| +25% | $E = 100; L = 10; S = 7.5; H = 18.75$<br>$E = (90, 95,100,105,110)$<br>$L = (9,9.5,10,10.5,11)$<br>$S = (6.5, 7,7.5,8,8.5)$<br>$H = (18.25,18.50,18.75,19,19.25)$ | 1.2507 | 1.2502 | 1.2499 | 1.2501 |
| +50% | $E = 120; L = 12; S = 9; H = 22.5$<br>$E = (110, 115,120, 125, 130)$<br>$L = (11, 11.5, 12, 12.5, 13)$<br>$S = (7, 8, 9, 10\ 11)$<br>$H = (21.5, 22, 22.5, 23, 23.5)$ | 1.1417 | 1.1403 | 1.1397 | 1.1401 |

## VI. Conclusion:

The parameters demand, holding cost, stockout cost and crashing cost are fuzzified by applying pentagonal fuzzy number. Defuzzification is done by using three methods such as Signed distance, Pascal triangular, graded mean and Graded Mean integration representation. Optimal Solution is found using Kuhn Tucker technique. Numerical example is studied and compared with various deffuzzification methods in both crisp and fuzzy senses are found that the values are almost equal.

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